Application of Barium Titanate Compositions to Parametric Amplification

This correspondence reports results of investigations of some of the problems associated with the application of ferroelectrics to microwave parametric amplification. The design of such an amplifier requires a knowledge of the variation of the permittivity of the material as a function of the electric field at microwave frequencies. Some information on the high frequency properties can be obtained by small signal incremental permittivity measurements at microwave frequencies. The material was used in the form of a cylindrical rod as this was convenient for preparation. The relative permittivity of a ferroelectric rod, placed in the center of a rectangular waveguide, and its variation with the applied dc field can be calculated from the measured values of the normalized reactance of the rod, the formulas for which are given by Marcuvitz [1]. Assuming that these formulas are valid also for lossless, high dielectric constant materials, then a rod of diameter d small compared to the width a of the guide can be represented by a shunt element whose normalized reactance for the dominant H_{01}-mode, is

$$\frac{X_a}{Z_0} \doteq \frac{a}{2\lambda g}\left[\frac{J_0(\psi)}{\psi J_1(\psi)} - S_0\right] \qquad (1)$$

where

$$\psi = \frac{\pi d}{\lambda}\sqrt{\epsilon_r'} \qquad (2)$$

and

$$S_0 = \ln\left(\frac{4a}{\pi d}\right) - 2 + \sum_{n=3,5,7}^{\infty}\left[\frac{1}{\sqrt{n^2 - \left(\frac{2a}{\lambda}\right)^2}} - \frac{1}{n}\right]$$

Let us say,

$$\frac{J_0(\psi)}{\psi J_1(\psi)} = f(\psi)$$

$f(\psi)$ passes through a series of resonances and antiresonances as the value of ψ is changed. The value of ψ at which antiresonance takes place, i.e., when $J_1(\psi)$ approaches zero, is not affected by the presence of S_0 which modifies the magnitude of $f(\psi)$. In the neighborhood of antiresonance, a small change in ψ gives rise to a large variation in the magnitude of $f(\psi)$ and as such in the magnitude of the reactance.

For materials with a complex relative permittivity $\epsilon_r = \epsilon_r' - j\epsilon_r''$ (2) becomes

$$\psi' = \frac{\pi d}{\lambda}\sqrt{\epsilon_r'}\sqrt{1 - j\frac{\epsilon_r''}{\epsilon_r'}}$$

Assuming that the ratio $\epsilon_r''/\epsilon_r' = \tan\delta$ is small, the normalized reactance of the rod is

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$$\frac{X_a}{Z} \doteq \mathrm{Re}\left[\frac{a}{2\lambda g}\left\{\frac{J_0(\psi e^{-j\theta})}{\psi[1-j\theta]J_1(\psi e^{-j\theta})} - S_0\right\}\right] \qquad (3)$$

$$\doteq \frac{a}{2\lambda g}\left[f_1(\psi) - S_0\right] \qquad (4)$$

where $\theta = \frac{1}{2}\tan\delta$.

For $\theta = 5° = 0.0873$ radians, i.e., $\tan\delta = 0.175$ (the choice was determined by the tables [2] available), $f_1(\psi)$ is plotted in Fig. 1. It is evident from Fig. 1 that to obtain the largest change in the reactance for a small change in the dielectric constant, the diameter of the specimen should be smaller than the first resonance value of ψ. For $\epsilon_r' = 5000$, $f = 9375$ Mc/s, the value of d for the first resonance value of ψ is nearly 0.5 mm and this was not possible to obtain. On the other hand, if barium titanate is mixed with a nonpolar binder as suggested by Cassidy, Jr. [3] the permittivity of the composite material could be controlled and one could expect to obtain the required value of ψ, initially, by trial and error.

Some preliminary measurements were made using solid rods of polycrystalline $BaTiO_3$ and composition rads made of the ground-up polycrystalline $BaTiO_3$ (Stanford University, Stanford, Calif., sample stated to have $\epsilon_r' = 5000$, $\tan\delta = 0.02$ and a 50 per cent change of ϵ_r' for a dc bias field of 20 kV/cm at S band) and polythene. The diameters were 0.199 cm (rod A), 0.054 cm (rod B), and 0.192 cm (rod C) for the solid rods and the composition respectively. Figure 2 shows the variation of the normalized shunt impedance of each rod measured as a function of frequency. The biasing wire was taken out through a narrow insulated opening on the narrow side of the guide. An applied dc bias of 8 kV/cm changed significantly the normalized reactance of rod C, but there was no significant change for the rods A and B or for any rods of intermediate diameters. Table I gives the relative permittivity of rod C using (1) and the most probable value of ϵ_r' is about 25.

Specimens of known compositions of powdered $BaTiO_3$ (Technical Ceramics Ltd., Northants, England, having a stated Curie temperature of 20°C) and polythene were made up. Measurements at a frequency of 850 kc/s showed that the zero-bias relative permittivity and loss tangent of the polycrystalline material were 6500 and 0.004, respectively. A biasing electric field of 8 kV/cm reduced the permittivity to 50 per cent of the zero bias value. Polythene (ICI) powder had a diameter of about 100 micron, but the diameter of the particles of powdered $BaTiO_3$ could not be controlled and varied over a large range of values. The compositions had a nominal diameter of two mm. The biasing copper wire or electrode was embedded into the center of the rod. The variation of the relative permittivity of the rods was measured as a function of the percentage of barium titanate in the composition at 3816 Mc/s using the E_{010}-mode cylindrical cavity technique developed by Horner et al. [4]. The measured Q (1750) of the empty cavity was much lower than the theoretical value (5500). The method used by Horner was used for calculating the loss factor of the specimens. The bias wire connected to the specimen picked up microwave power and for these reasons the results of Table II are somewhat approximate. To avoid these difficulties, the wire was removed and the specimen was reformed and remeasured in the cavity with the bias feed blanked off with a copper rod. The results, given in Fig. 3, show that a smooth variation of relative permittivity up to seventy can easily be obtained. It was very difficult to prepare specimens containing more than about 93 per cent of $BaTiO_3$.

The suitability of the composition rods as the variable reactance element in a parametric amplifier can be assessed in much the same way as can a variable capacitance diode. These experiments have already been described [5]. (This paper [5] gives a number of references which describe the properties of ferroelectrics and their applications at microwave frequencies.) Resonant cavity measurements were also made at X band to show directly which specimens gave an appreciable change of resonant frequency with applied dc bias. Figure 4 shows the resonance curves for a particular rod (90.3 per cent $BaTiO_3$) with and without bias. The ratio $\Delta f'/\Delta f$ is a measure of the quality factor of the composition.

The central biasing used so far is convenient for measurements with low level signals but unsuitable for parametric amplification. Let the specimen be centrally biased, so that the working points are at P and P' (Fig. 5). With signal, at any instant, the ac field increases the polarization of one half of the specimen and reduces the polarization of the other half. The changes in the incremental permittivity, due to the ac signal, in the two sections of the specimens are of opposite sign. Assuming these changes to be of equal magnitudes, they will cancel each other. For low level measurements, where one is primarily interested in ϵ_r' at the working point, and not in the change due to ac signal, central biasing is satisfactory. But for application to parametric amplification, where the change in the dielectric constant due to the signal is also important, central biasing is not useful. The output using central biasing system will be primarily at the second harmonic frequency. To avoid this the specimen is to be biased at one end. The rectangular cavity of the parametric amplifier is to be simultaneously resonant at three frequencies which are related by

$$f_p = f_i + f_s$$

To avoid the cavity resonances in the unwanted modes, the height of the cavity was kept small and the modes chosen were TE_{101}, TE_{301}, and TE_{303}. The pump power was obtained from a fixed frequency, 9420 Mc/s, pulsed X-band magnetron. The signal frequency was chosen in S band placing the idler in the C band. The breadth a of the cavity is

$$\left[\frac{f_p}{c}\right]^2 = \frac{9}{4a^2} + \frac{1}{4 \times 3.39a^2}$$

where c is the velocity of light.

The length of the cavity is given by $l = 1.843a$. The final dimensions of the cavity were 8.56 cm by 4.56 cm by 0.68 cm. The pump power was fed into the cavity from the input waveguide through a $\frac{1}{4}$ inch diameter coupling hole at the center of the plate 8.56 cm long. Unidirectional biasing

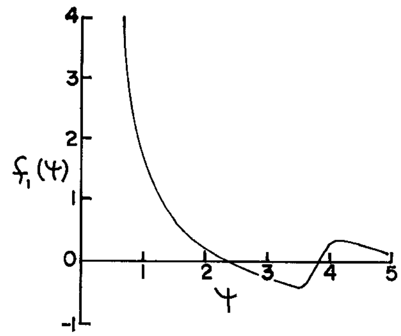

Fig. 1. Variation of $f_1(\psi)$ with ψ for $\theta = 0.0873$ radian, i.e., $\tan \delta = 0.175$.

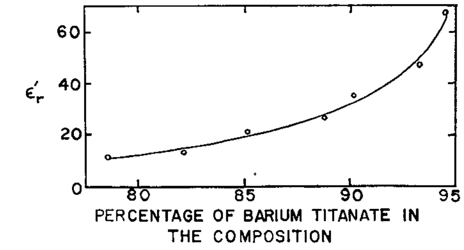

Fig. 3. Variation of ϵ_r' with percentage barium titanate in the compositions.

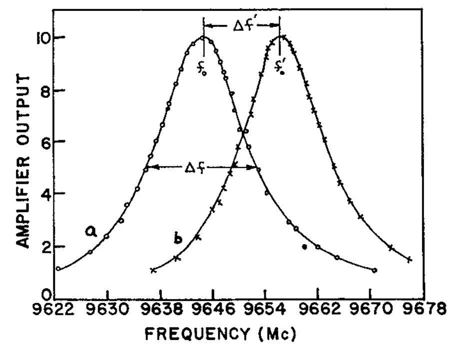

Fig. 4. Resonance curves of the waveguide cavity (a) Without bias (circles). (b) With a dc bias of 12 kV/cm (crosses).

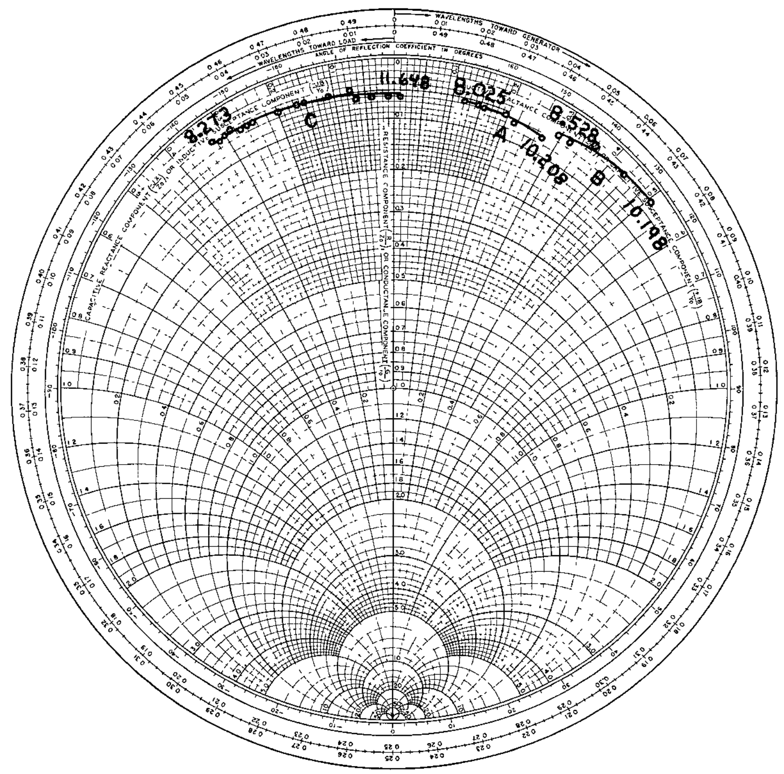

Fig. 2. Variation of normalized shunt impedance of the ferroelectric rods. Frequency is in Gc/s.

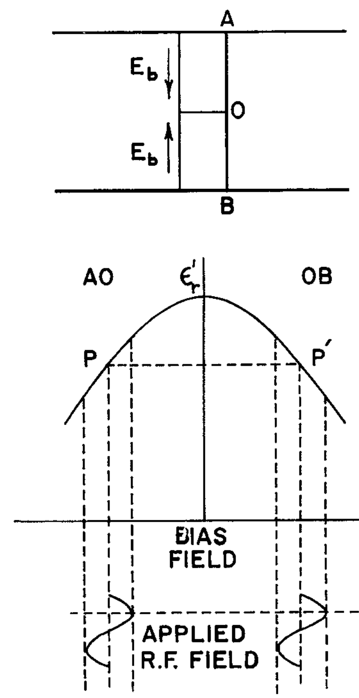

Fig. 5. Effect of central biasing.

TABLE I

Possible Values of Relative Permittivity of Specimen C at Different Frequencies

Frequency Mc/s	8273	8582	9090	9191	9985
ϵ_r'	24.9	24.8	26.5	26.6	27.4
	563	523	475	470	397
	1784	1681	1508	1483	1250
	3756	3496	3123	3081	2019

TABLE II

Variation of Dielectric Constant of Specimens with Applied DC Bias

Per cent of $BaTiO_3$	ϵ_r' Zero bias	Per cent decrease in ϵ_r'	dc bias (kV/cm)	1/(tan δ) (without biasing wire)
78.6	11.1	1	7.1	107
85.2	21.9	0.5	6.9	67
88.8	26.8	2.2	6.9	62
90.2	35.8	1.1	6.9	36

electric field was applied to the specimen. Typical values of the ratio $\Delta f'/\Delta f$ were 0.25 at f_p; 0.16 at f_i and 0.15 at f_s. These are lower than the values indicated below and was primarily due to low Q of the cavity. Figure 6 shows the experimental arrangement. By controlling the positions of M and N, the pump power could be controlled while the VSWR presented to the magnetron could be kept between 1.0 and 1.5. The coaxial low pass filters gave an isolation of more than 50 dB at the pump frequency. No trace of amplification was noted. With high average pump powers, for example 3 μs pulses at 250 PRF, the cavity has to be retuned to the signal frequency and the unamplified signal output increased by about 10 per cent. This was a temperature effect which disappeared with a PRF of 50. The calculated temperature rise in the rod is about 0.2°C per pulse and the center of the rod can be as much as 40°C above the room temperature.

A very rough estimation of the pump field inside the cavity can be done by assuming that all the power is dissipated and the whole of the energy is stored in the rod.

$$P = \frac{\omega W}{Q} = \omega W \tan \delta$$

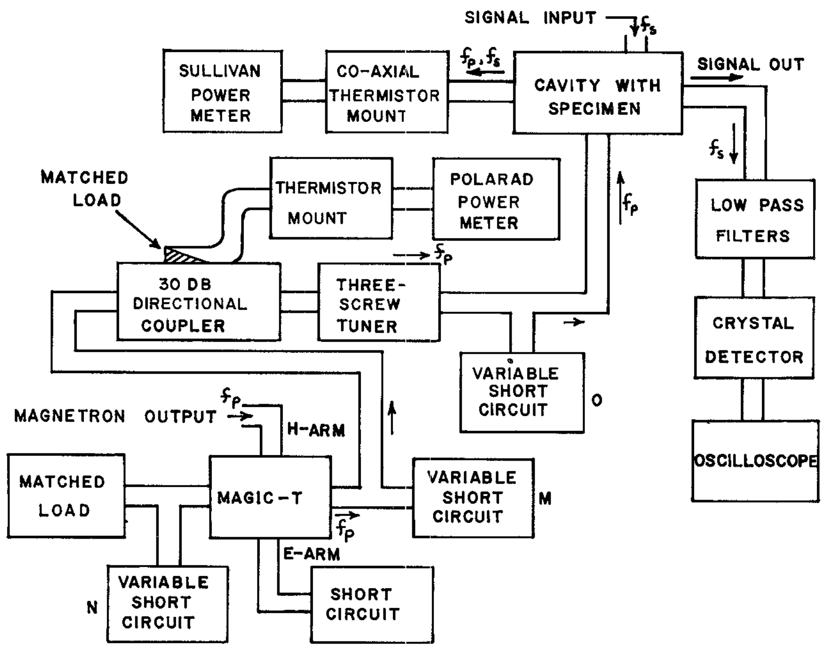

Fig. 6. Circuit arrangement for high power measurements.

where P is the power supplied and W is the energy stored. For a composition of 90.2 per cent $BaTiO_3$ of $\epsilon_r'=35$ and $\tan\delta=1/36$, the peak electric field is 7.8 kV/cm for an input power of 3.5 kW. The value of $\Delta f'/\Delta f$ necessary for large gain can be estimated as follows.

For large gain, the negative conductance presented to the signal circuit is equal to its conductance G_{T1}. Then [6]

$$G_{T1} = \frac{\omega_1\omega_2 C_3^2}{4G_2}.$$

Let C_p be the capacitance of the cavity at the pump frequency f_p and let ΔC be the capacity swing for the half-power points. Then

$$\frac{C_3}{\Delta C} = \frac{Q_p}{C_p}\sqrt{\frac{G_2 G_{T1}}{\omega_1\omega_2}} \tag{5}$$

where Q_p is the Q of the cavity at the pump frequency.

Beringer [7], considering the equivalent circuit of a lossless, unperturbed cavity to be a LC circuit, derives the following expressions for the equivalent capacitance C_a and inductance L_a of the cavity, $C_a=\epsilon/K_a^4 V$ and $L_a=\mu K_a^2 V$ where V = volume of the cavity and

$$K_a = \omega_a\sqrt{\mu\epsilon} \tag{6}$$

The cavity has a number of normal modes which are the periodic solutions of Maxwell's equations. Each of the modes is characterized by a resonant angular frequency ω_a.

$$\text{Let } f_3:f_2:f_1 = 3:2:1 \tag{7}$$

Substitution of (7) in (6) results in

$$C_p = \frac{4}{81}\sqrt{C_i C_s} \tag{8}$$

Substitution of this value of (8) in (5) gives us

$$\frac{C_3}{\Delta C} = 20.25\sqrt{\frac{Q_p}{Q_i Q_s}}$$

the subscripts i and s refer to the idler and signal frequencies, respectively. Assume $Q_p=Q_iQ_s$ then $\Delta f'/\Delta f=20.25$. For large gains approaching oscillation, the ratio of $\Delta f'/\Delta f$ should be 20, but some gain is to be expected when this ratio is lower than 20.

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A Plea for Clarity in Waveguide Designation

Apparently the use of letters to designate waveguide sizes [1] or frequency bands [2] is deeply ingrained in the microwave field. In addition, a number of standardization systems exist such that one may find an amazing number of different numbers and letters to describe a single size of waveguide. It may be noted in passing that the situation is worse for waveguide flanges.

In Table I, (pp 248–249) the International Electrotechnical Commission [3], British RCSC [4], American Military [5], American Civil EIA [6], and NATO [7], designation systems are tabulated along with the code letters used by American, British, European, and Japanese waveguide equipment manufacturers to designate the different sizes of rigid rectangular waveguide of approximately 2 to 1 dimension ratio.

The confusion caused by using letter designations is obvious while the different designation systems can hardly be said to contribute to clarity in specification. At this point it is felt that a plea should be entered for the IEC system: it has been worked out by an international organization and it is available in English and French for both English and metric measurement systems [3]. It is suggested that all specifications should carry the IEC designation in addition to any other system used. For example: "The instrument is available in RG-67/U (153 IEC-R 100 aluminum) waveguide." Such usage would certainly further international understanding in what is now a muddle of confusing waveguide designations.

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